DEPARTMENT OF PHYSICS
MAR THOMA COLLEGE FOR WOMEN, PERUMBAVOOR

SYMMETRIC COMPOUND PENDULUM

A symmetric compound pendulum is a type of pendulum that consists of multiple arms or rods attached to a central pivot point. Each arm has its own mass and length, and the arms are arranged symmetrically around the pivot point.

Aim

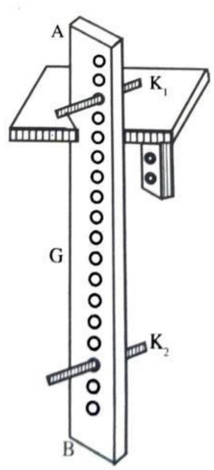
To calculate the acceleration due to gravity at a place, the length of the equivalent pendulum, the radius of gyration of the pendulum about an axis passing through the centre of gravity and perpendicular to its length and hence to calculate the moment of inertia by observing the oscillations of a compound pendulum.

Apparatus

The compound pendulum, two identical knife edges, rigid support, metre scale, stop clock and a weighing balance. The compound pendulum consists of a uniform rectangular bar (or cylindrical rod) AB of length one metre. A series of holes are drilled on it with their centres at equal distances. The bar can be suspended from a horizontal knife edge through anyone of the holes and can be made to oscillate in a vertical plane.

Theory

Consider a compound pendulum. Let S be the point of suspension of the pendulum through which passes a horizontal axis perpendicular to the length of the pendulum and G be the centre of gravity of the pendulum which is at a distance l from the point of suspension. Let the pendulum be displaced through a small angle, so that the centre of gravity shifted to G.



Then the couple r (torque) acting on the pendulum due to its weight mg is

$$r = Force \times Perpendicular\ distance\ (\ AG')$$

$$r = mgl\ sin\ \theta\ \ldots\ldots\ldots..(1)$$

This couple imparts an acceleration pendulum. If I be the moment of inertia of the pendulum about an axis passing through S and perpendicular to the length of the pendulum, the torque is given by

$$r = I\frac{d^2\theta}{dt^2}\ \ldots\ldots..\ (2)$$

According to Newtons third law, these two couples are equal and opposite. Thus we get

$$I\frac{d^2\theta}{dt^2} = -\ mgl\ \sin\theta$$

$$\text{Or}\ \ I\frac{d^2\theta}{dt^2} = -\ mgl\theta$$

$$(Since\ \sin\theta = \theta, when\ \theta\ is\ small)$$

$$\text{Or}\ \frac{d^2\theta}{dt^2} = -\ \frac{mgl}{I}\theta$$

This shows that the motion is simple Harmonic comparing the equation with the standard equation.

$$\frac{d^2\theta}{dt^2} = -\ \omega^2\theta$$

$$\text{We get}\quad \omega = \sqrt{\frac{mgl}{I}}$$

$$\text{Thus the time period T} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mgl}{I}}$$

But the moment of Inertia I of a pendulum is

$$I = I_g + ml^2$$

Where I_g is the moment of inertia of the pendulum about an axis passing through centre of gravity G

$$\text{i.e.,}\ I_g = mk^2$$

$$\therefore \; T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{mk^2 + ml^2}{mgl}} = 2\pi\sqrt{\frac{k^2+l^2}{gl}}$$

$$\text{Or} \qquad T = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

The quantity $\frac{\frac{k^2}{l}+l}{g}$ is called as the length of the equivalent (I_{eq}) simple pendulum. This can be found out from the graph between l and T

$$\text{Thus} \qquad T = 2\pi\sqrt{\frac{l_{eq}}{g}}$$

Squaring on both sides, we get

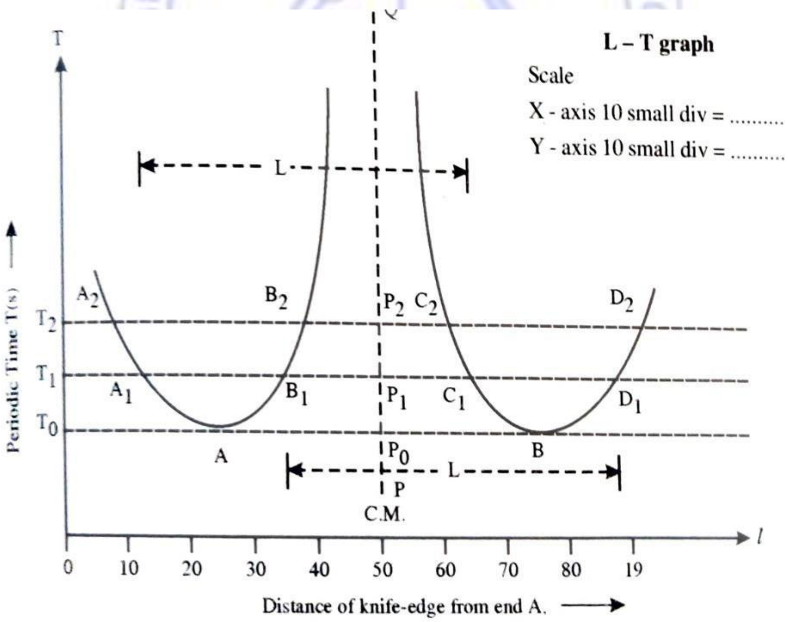
$$T^2 = 4\pi^2 \frac{l_{eq}}{g} \text{ or } g = 4\pi^2 \frac{l_{eq}}{T^2}$$

Using this acceleration due to gravity can be calculated.

Procedure

The pendulum is suspended using the knife edge. Insert the knife edge through the first hole near the end A. It is then placed on a rigid frictionless support so that the bar may oscillate freely in the vertical plane. The pendulum is pulled aside a little and then released. The pendulum oscillates in the vertical plane with small amplitude. The time for 20 oscillations is determined twice and the average time t isFound. Then the period of oscillation T is calculated.T=t/20

The experiment is repeated by suspending the pendulum from successive holes. The distances of holes where the knife edge touches are measured from the end. A. When the bar is suspended through the holes below the central hole, the bar is inverted but the distance is measured from the same end A. The centre of gravity of the bar is determined by balancing the bar on a knife edge. The distance of the centre of gravity from the end A is measured. A graph is drawn with the distances (1) from the end A along the X-axis and the Time period (T) along the Y-axis. We obtain two symmetric curves as shown in Figure. A line PQ is drawn parallel to Y-axis from the point corresponding to the distance of centre of gravity of the bar. The curves will be symmetric with respect to the line PQ.



Observations and tabulations

Distance of centre of gravity from the end A = ...m

To draw *l* versus T graph

Trial No.	Distance *l* of the knife edge from A (cm)	Time t for 20 oscillations in seconds			Period $T = \frac{t}{20}$ s
		t_1	t_2	Mean $t = \frac{t_1 + t_2}{2}$ (s)	
1					
2					
3					
.					
.					
.					

To find the length of the equivalent pendulum

To find the length of the equivalent simple pendulum, corresponding to any period (say T_1), a line is drawn parallel to the X-axis from the point corresponding to that period T_1. Let the line cut the curves at A_1, B_1, C_1 and D_1. The distances A_1C_1 and B_1D_1 are measured. The average of these two gives the length of the equivalent simple pendulum (l_{eq}). This is repeated for another time period T_2. This then gives the length of the equivalent pendulum for that period. This is repeated for five or six values of time periods and each time calculate $\frac{l_{eq}}{T^2}$. Mean value of $\frac{l_{eq}}{T^2}$ is evaluated. Then acceleration due to gravity can be calculated by using the formula $g = 4\pi^2 \frac{l_{eq}}{T^2}$.

To find the radius of gyration k

The distances A_1P_1, B_1P_1, C_1P_1 and D_1P_1 are measured from the graph. The radius of gyration of the compound pendulum about the axis through the centre of gravity and perpendicular to its length is given by

$$k = \sqrt{A_1P_1 \times C_1P_1} = \sqrt{B_1P_1 \times D_1P_1}$$

The mean value of k is calculated. This can be repeated for various horizontal lines and the average is taken.

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To find g and l_{eq}

Trial No.	Length l_{eq} in metre		Mean $l_{eq} = \frac{AC+BD}{2}$ (m)	Period T (seconds)	$g = 4\pi^2\left(\frac{l_{eq}}{T^2}\right)$
	AC (m)	BD (m)			

Mean acceleration due to gravity g = ...ms^{-2}

To find radius of gyration k

Serial No.	Distances in metre						$k = \sqrt{l_1 l_2}$ (m)
	AP (m)	PD (m)	Mean $l_1 = \frac{AP+PD}{2}$	BP (m)	PC (m)	Mean. $l_2 = \frac{BP+PC}{2}$	

The distance between the minima of the curves, AB = ...m

∴ $k = \frac{AB}{2}$ = ...m

∴ The mean value of k = ...m

Mass of the pendulum m = ...kg

Moment of inertia $I = mk^2$ = ...

Symme

Measure the distance AB, half of this gives k. i.e. $k = \frac{AB}{2}$

The mass m of the pendulum is found out by using a weighing balance. Using $I = mk^2$, the moment of inertia can be calculated.

Results

1. Acceleration due to gravity at the place = ... ms^{-2}
2. Radius of gyration of the pendulum about an axis passing through the centre of gravity = ...m
3. Moment of inertia of the pendulum about an axis passing through the centre of gravity and perpendicular to its length, I = ... kgm^2

Note : The length l of the pendulum should always be measured from the same end A.

Working formula

$$g = 4\pi^2 \left(\frac{l_{eq}}{T^2}\right)$$

$$I = mk^2$$

$$k = \sqrt{l_1 l_2}$$

Reference

Experimental Physics – I, For First, Second, Third and Fourth Semester, BSc Degree Programme, Dr.P.Sethumadhavan, Prof. K.C. Abraham, Prof .Sunil John, **Manjusha Publications**

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